Exact Analysis of Maximum Available Gain and Unilateral Gain Including Phase Angle of $S21$

Yu-Hsun Huang, *Student Member, IEEE*, Chi-Chung Chien, and George D. Vendelin, *Fellow, IEEE*

***Abstract*—Exact formulae are presented for both maximum available gain (G_{ma}) and unilateral gain (U), which include both the magnitude and phase angle of the S_{21} parameter. The result for G_{ma} is not unique since there are many possible solutions to simultaneously match S_{11} and S_{22} when $k > 1$. For the phase angle of U, the result tends to become 180° when using the topology of a variable coupler, a line stretcher, and a feedback amplifier. From the verifications, an amplifier using the unilateralizing technique will achieve 4–6 dB higher gain than that of the G_{ma} amplifier.**

***Index Terms*—Amplifiers, feedback, microwave circuits.**

I. Introduction

FOR AMPLIFIER design, various power gains of a stable circuit have been defined in the past 50 years [1]. Unilateral gain (U) defined by Mason [2] is the highest gain of these definitions and is realized by the unilateralization technique. The technique discussed by Cheng [3] shows several different networks to unilateralize a circuit. An approach to measure unilateral gain was proposed by Lange [4], which reveals that it is possible to build a stable amplifier with U. A more recent review paper about unilateral gain was published by M. Gupta [5]. To design a stable unilateral amplifier, an exact analysis of the gain and the active device is necessary.

The gain formulae, G_{ma} and U, only provide the magnitude. The phase angle of S_{21} is not available from the previous analysis. In this letter, exact G_{ma} and U amplifier design equations with the phase angle of S_{21} are derived. Further study of the amplifiers with unilateral gain has shown that a phase angle of S_{21} approaching 180° is possible, which allows a perfect inverter. A 180° phase shift signal has many future applications at frequencies of 18 GHz or higher. The design of the G_{ma} amplifier and five different types of stable unilateral amplifiers are presented and verified by commercially-available simulation software such as Agilent ADS (Advanced Design System), Applied Wave Research Microwave Office, and Ansoft Serenade.

II. Maximum Available Gain

The following analysis is based upon [1] (problem 1.17). If two two-ports, S_m and S_n, are cascaded, the resulting two-port has the following S-parameters:

$$S_{11}=S_{m11}+\frac{S_{m12}S_{m21}S_{n11}}{1-S_{m22}S_{n11}} \qquad S_{12}=\frac{S_{n12}S_{m12}}{1-S_{m22}S_{n11}}$$
$$S_{21}=\frac{S_{n21}S_{m21}}{1-S_{m22}S_{n11}} \qquad S_{22}=S_{n22}+\frac{S_{n12}S_{n21}S_{m22}}{1-S_{m22}S_{n11}}. \quad (1)$$

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The authors are with Santa Clara University, Santa Clara, CA 95053 USA (e-mail: yuhsun.huang@ieee.org; ccchien74@yahoo.com; gvendelin@yahoo.com).

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The G_{ma} amplifier is the cascade of three two-ports. By the above formulae and the assumption of a perfect input and output match $S_{11}=S_{22}=0$, the result shown as below:

$$S_{21}=\frac{S_{g21}S_{m21}S_{n21}}{(1-S_{g22}S_{m11})(1-S_{m22}S_{n11})-S_{g22}S_{m21}S_{m12}S_{n11}} \quad (2)$$

$$S_{12}=\frac{S_{g12}S_{m12}S_{n12}}{(1-S_{g22}S_{m11})(1-S_{m22}S_{n11})-S_{g22}S_{m21}S_{m12}S_{n11}}. \quad (3)$$

This result gives both the gain and phase of S_{21}, but the phase depends on the particular matching structure. These formulae for S_{21} and S_{12} have been verified in ADS, Microwave Office, and Serenade. Equation (2) for S_{21} bears a startling resemblance to the transducer gain equation, G_T [1].

III. Unilateral Gain

The unilateral gain, which is defined by Mason, has usually been used as a design criterion for transistors, because the gain is invariant to the common terminal for an active device [5]. To achieve the unilateral gain, S_{12}, S_{11}, and S_{22} of the four S-parameters are reduced to zero using lossless feedback and lossless matching regardless of the value k. The first method of unilateralizing an amplifier is to use two lossless feedback elements to cancel the real and imaginary parts of S_{12}. This method will turn out an arbitrary angle of S_{21}. Three examples of this method with different feedback circuits are given in Fig. 1.

Another unilateralizing method uses a variable 90° coupler for the lossless feedback, as proposed by Lange [4], is given in Fig. 2, which consists of a unilateralizing variable coupler, a line stretcher, and an internal amplifier that is a G_{ma} amplifier. The S-parameters are given in the following.

A) The S-parameters of an ideal directional coupler are

$$\mathbf{S}_{lc}=\begin{bmatrix}0 & C & -jT & 0\\ C & 0 & 0 & -jT\\ -jT & 0 & 0 & C\\ 0 & -jT & C & 0\end{bmatrix} = \begin{bmatrix}0 & C & T\angle-90^\circ & 0\\ C & 0 & 0 & T\angle-90^\circ\\ T\angle-90^\circ & 0 & 0 & C\\ 0 & T\angle-90^\circ & C & 0\end{bmatrix} \quad (4)$$

with $C^2=1-T^2$, where port 3 is the through port and port 2 is the coupled port.

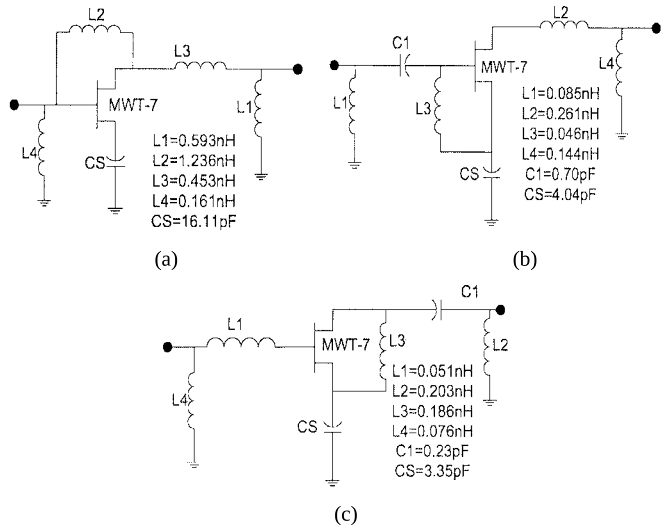

Fig. 1. (a) RF schematic diagram of Circuit A and shunt-shunt and series-series feedbacks [3]. (b) The RF schematic diagram of Circuit B with a series-shunt feedback [3]. (c) The RF schematic diagram of Circuit C with a shunt-series feedback [3].

B) The S-parameters of a G_{ma} amplifier are defined by

$$\mathbf{S}_{ma} = \begin{bmatrix} S_{11\_ma}\angle\theta_3 & S_{12\_ma}\angle\theta_2 \\ S_{21\_ma}\angle\theta_1 & S_{22\_ma}\angle\theta_4 \end{bmatrix},$$

$$|\mathbf{S}_{21\_ma}| = \sqrt{G_{ma}} > 1 \quad \text{and} \quad |\mathbf{S}_{12\_ma}| = \sqrt{G_{maR}} < 1 \quad (5)$$

where $G_{maR} = |\mathbf{S}_{12}/\mathbf{S}_{21}|\,(k - \sqrt{k^2-1})$. G_{maR} is the reverse G_{ma}.

C) The S_{12} and S_{21} of the lossless line stretcher are $\mathbf{S}_{ls\_12} = 1\angle\phi$ and $\mathbf{S}_{ls\_21} = 1\angle\phi$.

D) The derivations of S-parameters of a unilateral amplifier with 90° directional coupler (Fig. 2) are shown by Huang [7]. The results are shown in the following:

$$\mathbf{S}_{11} = \frac{b_1}{a_1} = T^2 S_{11\_ma}\angle(-180^\circ + \theta_3) \quad (6)$$

$$\mathbf{S}_{12} = \frac{b_1}{a_2} = \frac{C\angle 0^\circ - \sqrt{G_{maR}}\angle(\phi+\theta_2)}{1 - C\sqrt{G_{maR}}\angle(\phi+\theta_2)} \quad (7)$$

$$\mathbf{S}_{21} = \frac{b_2}{a_1} = \frac{C\angle 0^\circ - \sqrt{G_{ma}}\angle(\phi+\theta_1)}{1 - C\sqrt{G_{ma}}\angle(\phi+\theta_1)} \quad (8)$$

$$\mathbf{S}_{22} = \frac{b_2}{a_2} = T^2 S_{22\_ma}\angle(-180^\circ + \theta_4 + 2\phi). \quad (9)$$

Consider the measurement of S_{12} using Fig. 2, where the input signal is a_2 at the output port and the reflected signal is b_1 at input port. For S_{12}, the line stretcher varies the phase such that the coupled portion of a_2 is 180° out of phase with the portion of a_2 from the transistor amplifier and the line stretcher. If S_{12} draws near to zero to achieve unilateralization, the value of S_{21} is that of the unilateral gain of the transistor amplifier. The derivation of unilateralization is shown by Huang [7] and the result is

$$\mathbf{S}_{12} = 0, \text{ only if } \phi + \theta_2 = 0^\circ \quad \text{and} \quad C = \sqrt{G_{maR}}. \quad (10)$$

This approach will also produce an arbitrary phase angle of S_{21}. Circuit D in Fig. 3 uses the approach and the simulation

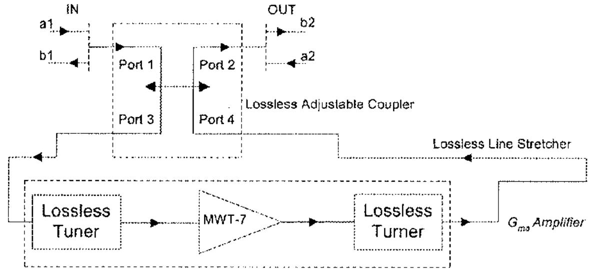

Fig. 2. Schematic diagram of the unilateral amplifier.

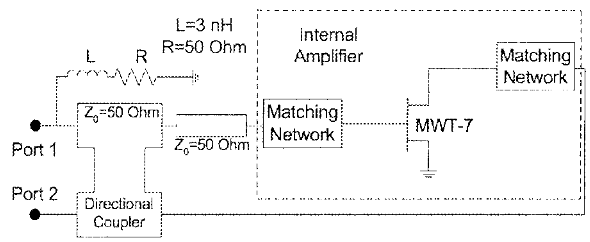

Fig. 3. RF schematic diagram of Circuit D.

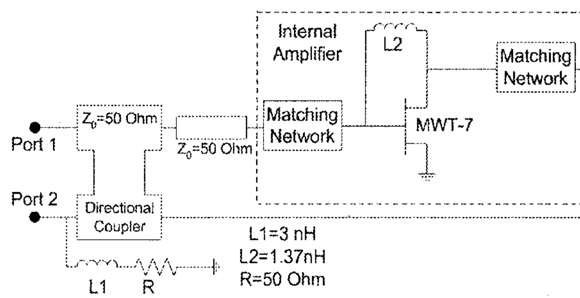

Fig. 4. RF schematic diagram of Circuit E.

results are listed in Table II. Equation (10) is a general analysis for the circuit shown in Fig. 4. Then, a special condition is discussed as following. From [7]:

$$\mathbf{S}_{21} = -\sqrt{U} \text{ if } \phi + \theta_2 = 0^\circ. \quad (11)$$

Consequently, a perfect inverter at microwave frequencies can be built when the circuits satisfy (7), (8), (10), and (11), which means $\theta_1 = \theta_2$. A novel circuit topology, Circuit E shown in Fig. 4, that fulfills (10) and (11) is proposed in this paper to achieve a unilateral amplifier with 180° of S_{21}. The internal amplifier consists of a CE transistor, matching networks, and a parallel-parallel feedback network, which makes the phase angles of S_{21} and S_{12} of the internal amplifier the same. This result also includes both magnitude and phase for the unilateral gain, which is verified on ADS.

IV. Verification

MWT-7 MESFET from Microwave Technology Inc., where the bias conditions are $V_{ds} = 5$ V and $I_d = 28$ mA, is used in circuits A, B, C, D, and E for the same operational frequency, 18 GHz. The k is 1.117 and the G_{ma} is 10.32 dB. The theoretical U is 15.47 dB from Mason's equation. Using the lossless feedback elements to unilateralize the MESFET, the three topologies in Fig. 1 give the required unilateral gain and various phase angles of S_{12} and S_{21}. The results are shown in Table I. Table II

TABLE I
S-PARAMETERS OF MWT-7, CIRCUIT A, B, AND C AT 18 GHz

	Circuit D		Circuit E	
	Without the parallel R and L at port 1	With the parallel R and L at port 1	Without the parallel R and L at port 2	With the parallel R and L at port 2
C (dB /Deg)	-14.49 / 0°	-14.50 / 0°	-32.92 / 0°	-33.69 / 0°
ϕ (Deg)	-122.00°	-120.92°	-100.30°	105.70°
S_{11} (dB/Deg)	-30.04 / -30.01°	-30.20 / 72.95°	-28.68 / -19.88°	-26.54 / -38.01°
S_{12} (dB/Deg)	-84.00 / -54.38°	-67.88 / -132.45°	-80.00 / 163.14°	-70.10 / 143.23°
S_{21} (dB/Deg)	15.47 / -120.21°	15.35 / -116.04°	15.46 / -179.94°	15.35 / -179.48°
S_{22} (dB/Deg)	-30.03 / 104.96°	-30.66 / 137.19°	-34.04 / 26.21°	-29.94 / 38.76°
Internal amplifier's $S_{12}\angle\vartheta_2$ (dB /Deg)	-14.49 / 121.99°	-14.50 / 121.97°	-32.89 / 100.26°	-33.58 / 105.29°
Internal amplifier's $S_{21}\angle\theta_1$ (dB /Deg)	10.32 / 148.49°	10.32 / 147.47°	14.40 / 100.39°	14.46 / 102.37°

TABLE II
PERFORMANCE OF CIRCUIT D AND CIRCUIT E AT 18 GHz

	S_{11} (dB/Deg)	S_{12} (dB/Deg)	S_{21} (dB/Deg)	S_{22} (dB/Deg)
MWT-7	-2.81 / -155.90°	-20.00 / 15.50°	4.81 / 42°	-8.57 / -89.60°
Circuit A	-28.33 / 0.14°	-50.99 / -86.95°	15.61 / 91.77°	-60.01 / -21.80°
Circuit B	-59.81 / -170.09°	-50.99 / 87.97°	15.62 / -91.98°	-53.91 / 148.37°
Circuit C	-41.80 / 69.84°	-50.99 / 82.61°	15.62 / -96.95°	-59.94 / -69.17°

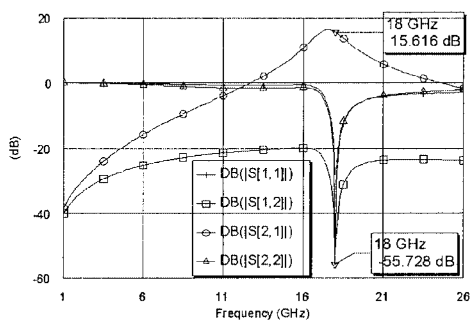

Fig. 5. S-parameters of Circuit A.

illustrates the simulation results of Circuit E and Circuit D. Circuit E's θ_1 is equal to θ_2. The phase angle for circuit E is 180° for S_{21}, which fits (11).

In Fig. 3 and Fig. 4, the parallel resistors in the port 1 and port 2 are used to make the two circuits stable but decrease S_{21} by 0.1 dB. The behaviors with the frequency of the five circuits are similar, and Fig. 5 plots the S-parameters of Circuit A. The five values of S_{21} in dB approach the theoretical unilateral gain and the 3 dB bandwidths of S_{21} are typically 10%. All circuits are designed to be stable by the topology of the shunt inductor of the input and output matching networks. Using the stability circles to check these cases, the Circuit A, B, C, D, and E are stable from 1 GHz to 18 GHz, even $k < 1$, which means the circuits are conditionally stable.

In the five circuits, Circuit A is the most realizable one to achieve U. Circuit D is more complex, but a microwave inverter, Circuit E, is deduced from Circuit D.

V. CONCLUSION

The design of amplifiers for G_{ma} and U has been presented in a new form. Once the G_{ma} amplifier is determined, the unilaterilized amplifier may be achieved with a coupler and line stretcher, which is demonstrated by Circuit D. The value of the coupling factor of the coupler is simply the $|S_{12}|$ of the G_{ma} amplifier. A new topology is proposed (Circuit E) which makes the phase angle of U approximately 180° will have many applications in microwave circuits. The equations for the new specified topology give both the magnitude and phase of the S_{21}, which is another new feature in the result. From the results of this paper, it is possible to design an amplifier with unilateral gain and an arbitrary phase angle of S_{21}. The experimental results to verify the advantages of unilateral amplifiers will be published soon [6].

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